Scheduling for Weighted Flow and Completion Times in Reconfigurable Networks

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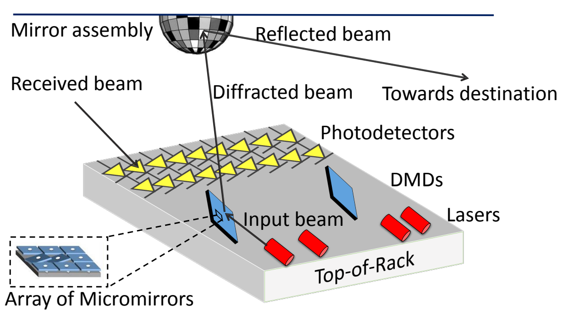
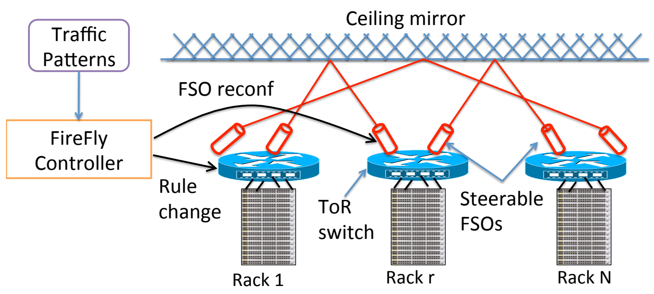
Reconfigurable Networks

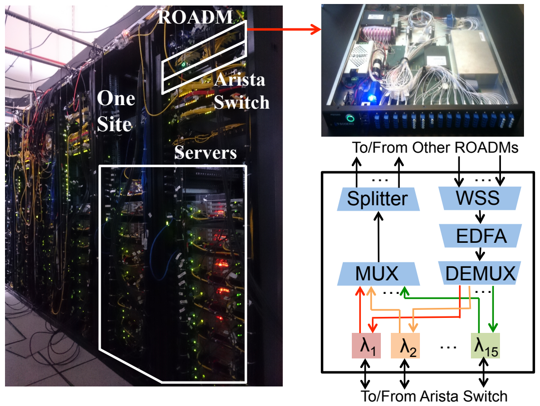
Can change network topology in software!

Datacenters





Optical WANs



(a) Testbed. (b) ROADM.

Many constraints depending on technology

Always: degree bounds

Reconfiguration Can Be Helpful

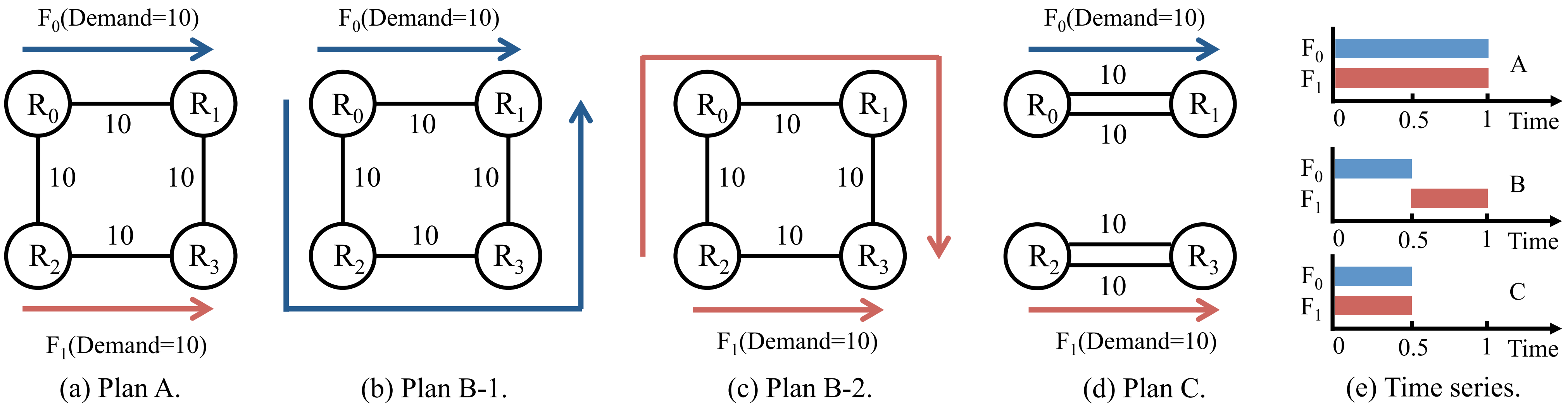


Image and example from Jin et al, SIGCOMM '16

Scheduling Bulk Transfers

System:

- Optimizing Bulk Transfers with Software-Defined Optical WAN [Jin et al. SIGCOMM ‘16]

Theory:

- Competitive Analysis for Online Scheduling in Software-Defined Optical WAN [Jia et al. INFOCOM ‘17]

Given bulk transfers (online), how should we schedule transfers & reconfigurations?

Model [Jia et al.]

Start:

- Nodes V, degree bounds d_v for each $v \in V$
- Transfers (jobs) S

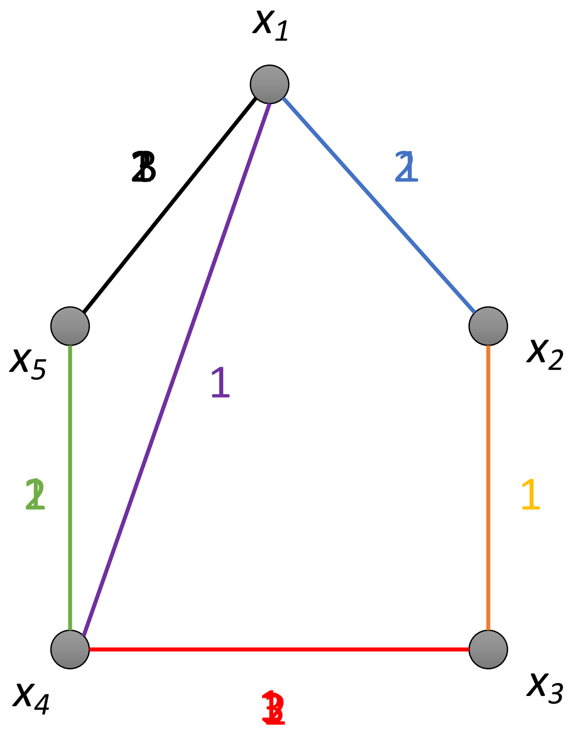
Transfer (job) i:

- Release time r_i, source u_i, destination v_i, size l_i, weight w_i (not in Jia et al)

Time t:

- Create graph $G_t = (V, E_t)$ obeying degree bounds
  - E_t subset of transfers S
- One unit of progress on jobs in E_t

Example



$d_v = 1$ for all v

Transfer	Release	Source	Destination	Size
1	1	x_1	x_5	3
2	1	x_1	x_2	2
3	1	x_2	x_3	1
4	2	x_5	x_4	2
5	2	x_4	x_3	3
6	4	x_1	x_4	1

Issues with Model

- No constraints on graphs other than degrees
  - Optical WANs: real constraints based on optical network
  - Datacenters: depending on technology

- Can only send data over direct connections
  - OWAN system uses multihop paths

Still a good start!

Objectives and Results (Jia et al)

Given schedule, each transfer *i* has **completion time** C_i

Makespan

- $max_i\ C_i$
- Time when last job completes
- 3-competitive algorithm

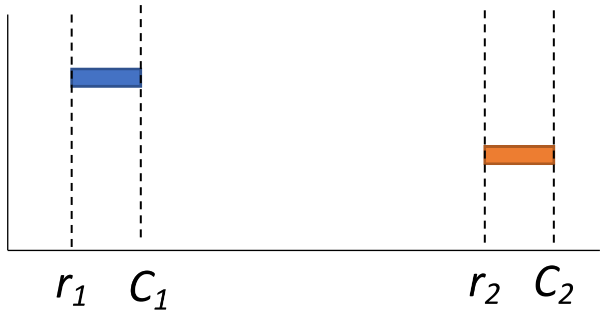
Sum of Completion Times

- $\sum_i C_i$
- 3α-competitive algorithm
  - α competitive ratio of SRPT for d-machine scheduling
  - At most 1.86
  - Assumes $d_v = d$ for all v

α-competitive: at most α factor worse than offline optimum

Flow Time

In online setting, do these objectives make sense?



Makespan unchanged, sum of completion times only doubled!

New Objective: Sum of (Weighted) Flow Times

- Flow time of job *i*: $F_i = C_i - r_i$
- Sojourn time, waiting time, response time
- $\sum_i w_i(C_i - r_i)$

Our Results:

Lower bound: Every online algorithm has competitive ratio at least $\Omega(\sqrt{n})$

Upper bound: need **resource augmentation / speedup**

- Allow faster transfer compared to OPT
  - Our solution uses 200 Gbps links, compare to OPT using 100Gbps links
- *$O(1/\varepsilon^2)$*-competitive algorithm with *$(2+\varepsilon)$*-speedup

Corollary: *O(1)*-competitive algorithm for **weighted** sum of completion times, **different** degree bounds (no speedup)

Algorithm: Highest-Density First

- Density of job i: $h_i = \frac{w_i}{l_i}$
- At time *t*:
  - Order jobs in nonincreasing order of density
  - Schedule job *i* (add $u_i - v_i$ edge) if u_i and v_i not already full

Easy to state, tricky to analyze!

- Reduce to unit-length jobs (via "fractional" flow time): cost $O(1/\varepsilon)$
- Dual Fitting: cost $O(1/\varepsilon)$

LP relaxation (unit length)

$$\min \sum_{i \in S} \sum_{t \geq r_i} w_i (t - r_i) x_{i,t}$$

Weighted flow time

$$\text{s.t.} \quad \sum_{t \geq r_i} x_{i,t} \geq 1 \qquad \forall i \in S$$

Every job gets scheduled

$$\sum_{i \in S : |\{u_i, v_i\} \cap \{w\}| = 1} x_{i,t} \leq d_w \qquad \forall w \in V, \ \forall t \in \mathbb{N}$$

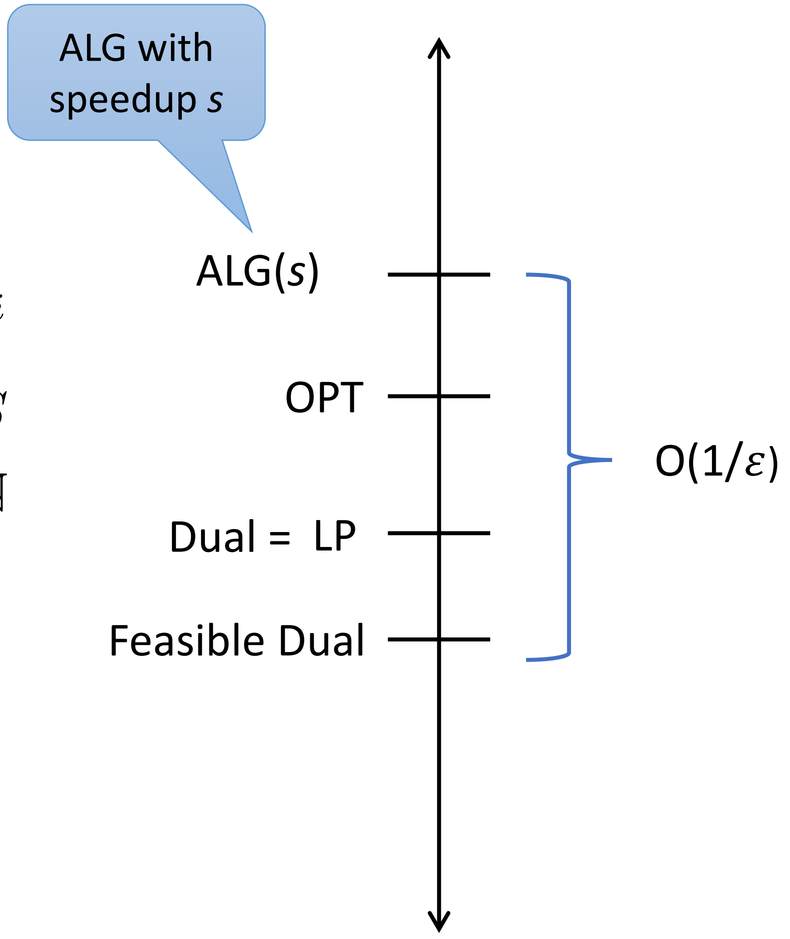
Degree bounds

$$x_{i,t} \geq 0 \qquad \forall i \in S, \ \forall t \in \mathbb{N}$$

1 if job *i* scheduled at time *t*

Dual

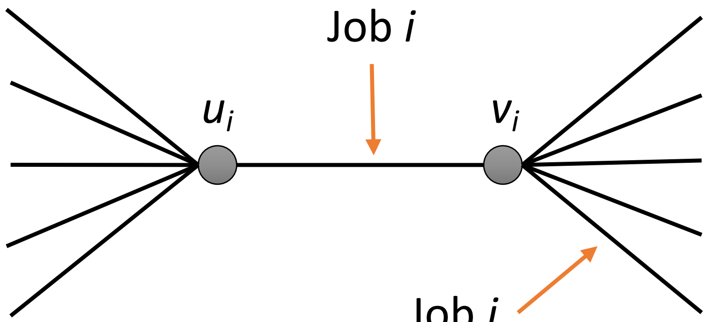
$$
\begin{aligned}
\max \quad & \sum_{i\in S} \alpha_i - \sum_{u\in V}\sum_{t\in\mathbb{N}} \beta_{u,t} \\
\text{s.t.} \quad & \alpha_i - \frac{\beta_{u_i,t}}{d_{u_i}} - \frac{\beta_{v_i,t}}{d_{v_i}} \le w_i(t-r_i) && \forall i\in S,\ \forall t\ge r_i \\
& \alpha_i \ge 0 && \forall u \in S \\
& \beta_{i,t} \ge 0 && \forall i\in S,\ \forall t\in\mathbb{N}
\end{aligned}
$$



- Dual fitting: common in flow time scheduling problems
- Intuition:
  - α_i = increase in algorithm's cost due to transfer *i* when it is released
  - $\beta_{u,t}$ = remaining work at node *u* at time *t*

Dual Solution: α

α_i = increase in algorithm's cost due to transfer i when it is released



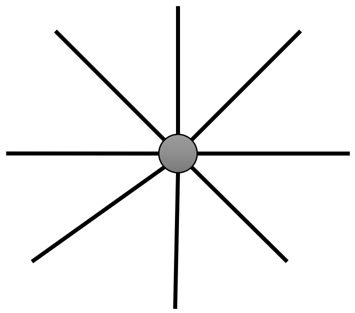
Job j with $w_j > w_i$: scheduled before $i \Rightarrow$ increase in total weighted flow is w_i

Job j with $w_j < w_i$: scheduled after $i \Rightarrow$ increase in total weighted flow is w_j

$$\alpha_i := \frac{1}{2s}\left(\frac{1}{d_{u_i}}\left(\sum_{j\in U_i(r_i):w_i<w_j} w_i + \sum_{j\in U_i(r_i):w_i>w_j} w_j\right) + \frac{1}{d_{v_i}}\left(\sum_{j\in V_i(r_i):w_i<w_j} w_i + \sum_{j\in V_i(r_i):w_i>w_j} w_j\right)\right)$$

Dual Solution: β

$\beta_{u,t}$ = remaining work at node u at time t



$$\beta_{u,t} = \frac{w_u(t)}{2s}$$

Total weight of jobs at u at time t

Speedup $(2+\varepsilon)$

Main Result

$$\max \quad \sum_{i\in S} \alpha_i - \sum_{u\in V}\sum_{t\in\mathbb{N}} \beta_{u,t}$$

$$\text{s.t.} \quad \alpha_i - \frac{\beta_{u_i,t}}{d_{u_i}} - \frac{\beta_{v_i,t}}{d_{v_i}} \leq w_i(t-r_i) \qquad \forall i \in S,\ \forall t \geq r_i$$

$$\alpha_i \geq 0 \qquad \forall u \in S$$

$$\beta_{i,t} \geq 0 \qquad \forall i \in S,\ \forall t \in \mathbb{N}$$

Feasibility: $\alpha_i - \frac{\beta_{u_i,t}}{d_{u_i}} - \frac{\beta_{v_i,t}}{d_{v_i}} \leq w_i(t-r_i)$

Lemma: $\sum_{i\in S}\alpha_i \geq \frac{1}{2} ALG(s)$

Lemma: $\sum_{u\in V}\sum_{t\in\mathbb{N}}\beta_{u,t} \leq \frac{1}{s} ALG(s)$

Theorem: There is a feasible dual solution with value at least

$$\frac{\varepsilon}{2\varepsilon+4} ALG(2+\varepsilon)$$

Conclusion & Open Questions

Our work:

- Model of scheduling transfers in reconfigurable networks from Jia et al. [INFOCOM ‘17]
- In online setting, flow times make more sense than completion times
- First nontrivial approx for flow times, with small speedup (necessary)
- Corollary: first O(1)-competitive algorithm for completion times

Future work:

- More realistic model of reconfigurable networks!
- Speedup 1+ε instead of 2+ε?

Thanks!