Minimum Congestion Routing of Unsplittable Flows in Data-Center Networks

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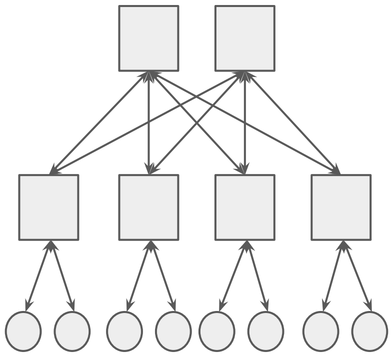
Most data-centers are modeled after Clos networks

[Singh *et al.* 15, Greenberg *et al.* 15, Gangidi *et al.* 24, Qian *et al.* 24]



Folded Clos network

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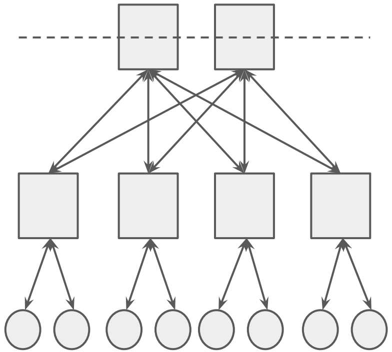
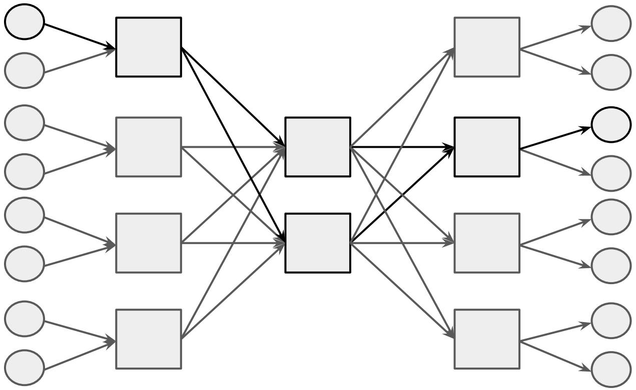


Unfolded around symmetry axis



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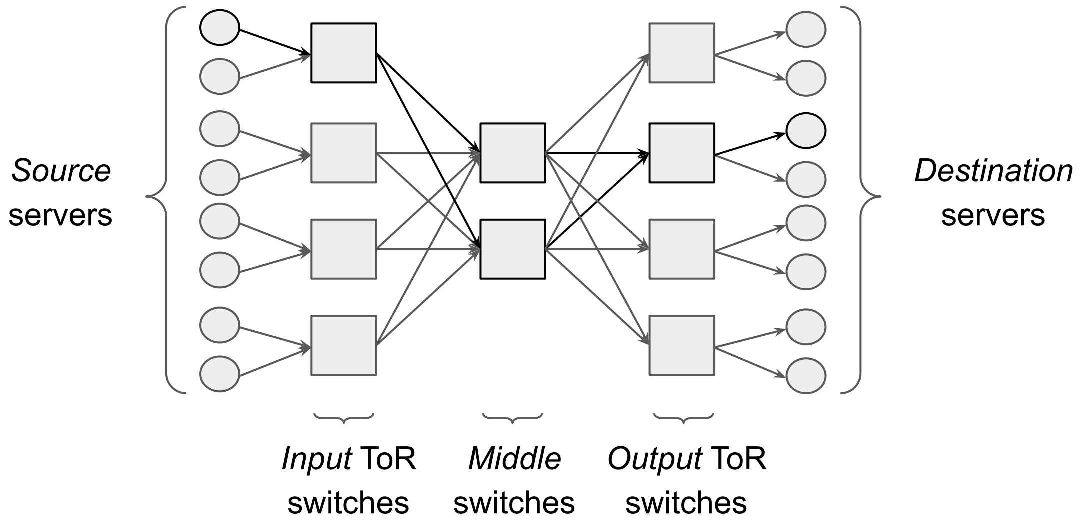
Characteristics of Clos networks [Clos 53]



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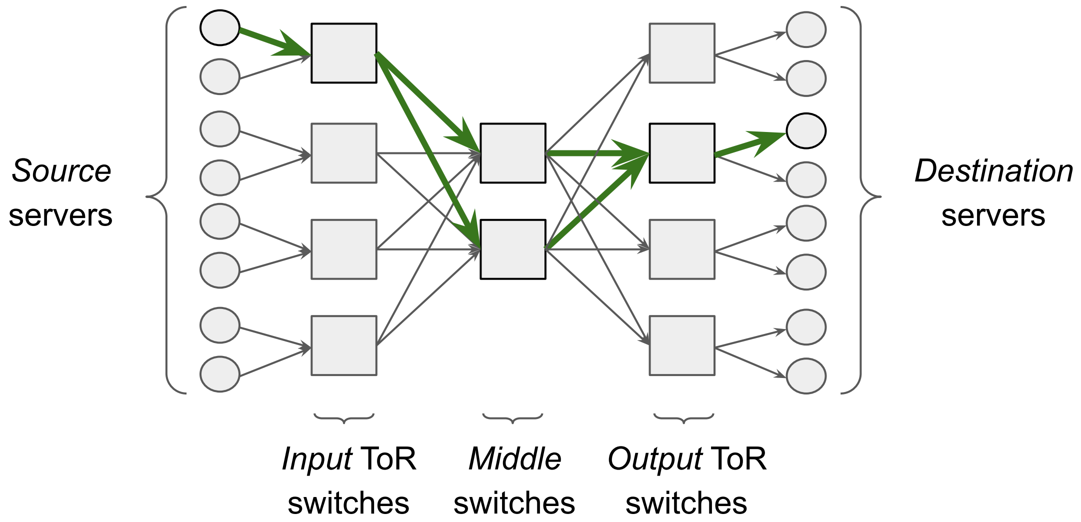

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- N = number of servers per ToR switch
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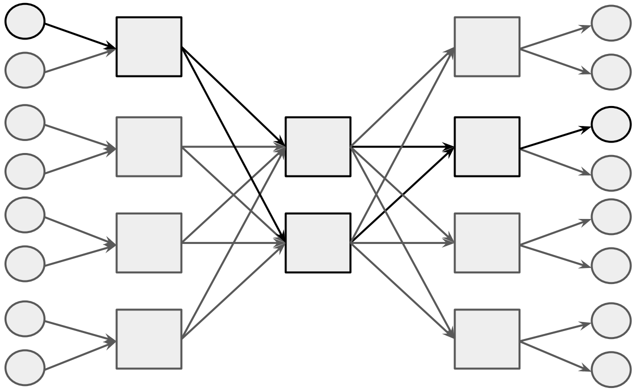

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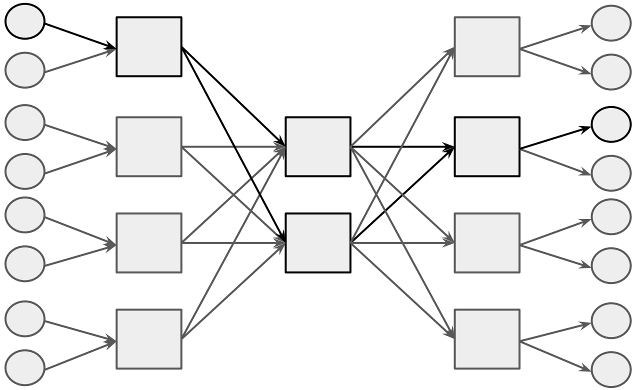
- Uniform link capacities



Data-center wish to closely satisfy flow demands …

[Ballani *et al.* 11, Lee *et al.* 14]

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… therefore, flow routing in data-center seeks to minimize congestion [Alizadeh *et al.* 14, Singla *et al.* 14, Namyar *et al.* 21]

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Every flow satisfies its demand if and only if routing has congestion ≤ 1

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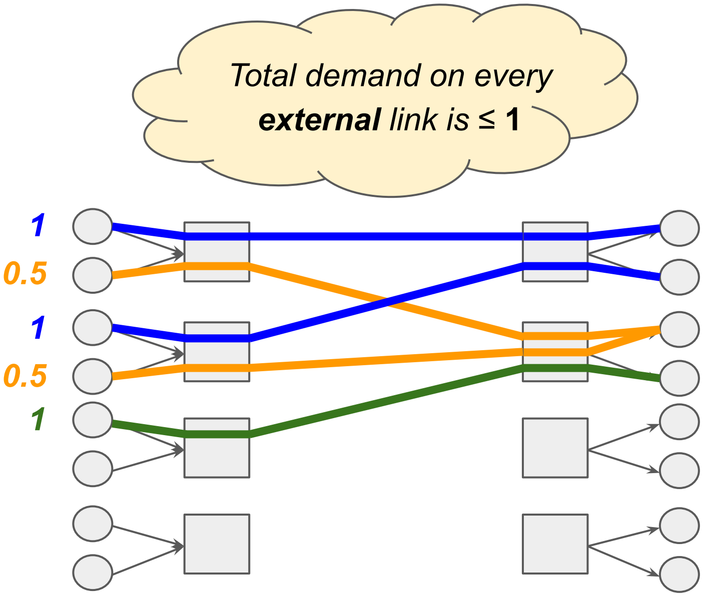

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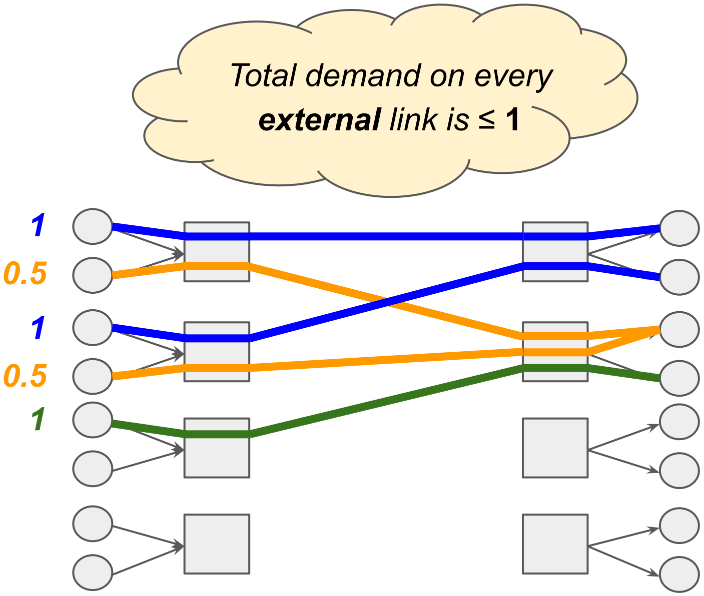

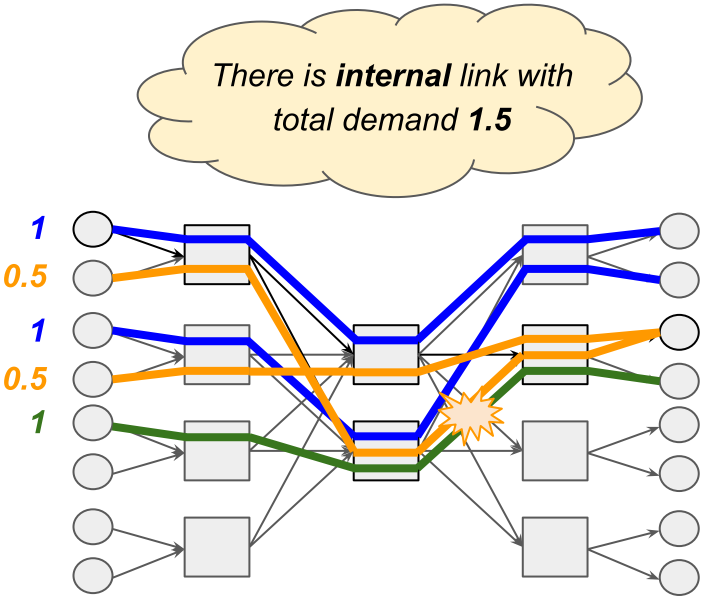

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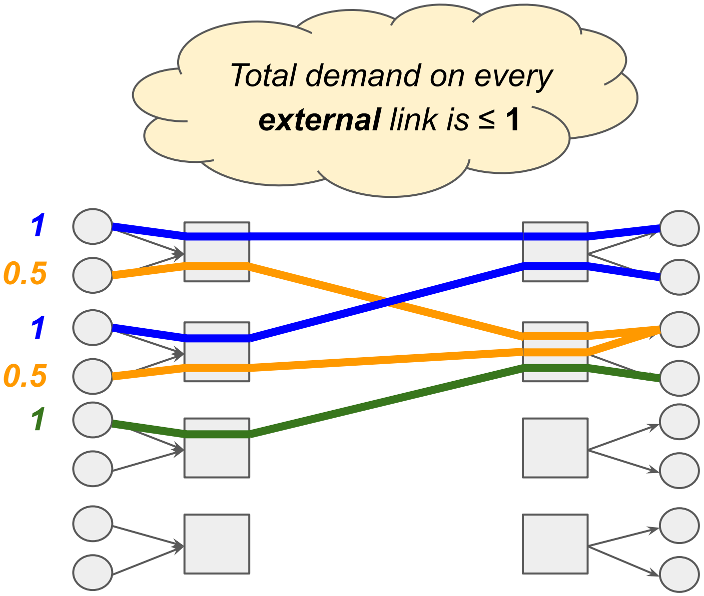

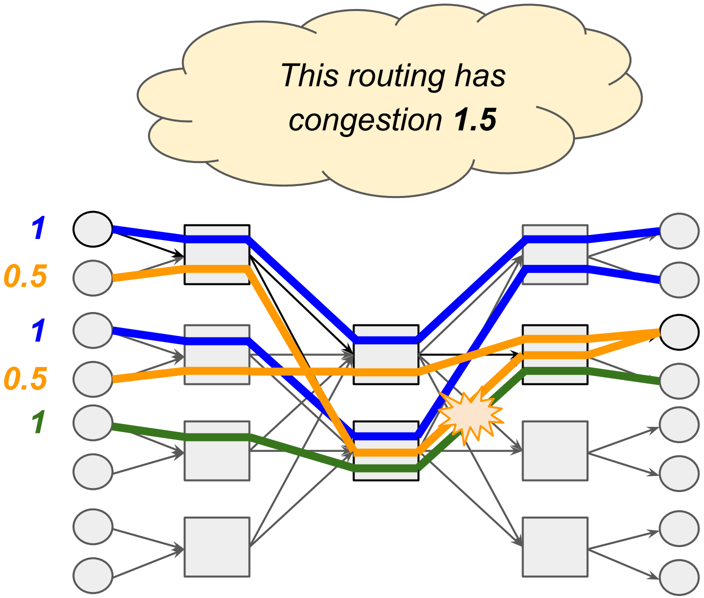

With arbitrary flow splitting, minimizing congestion is easy

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With splittable flows, every flow satisfies its demand

However, flows splitting has significantly deployability challenges

[Qureshi *et al.* 22, Gangidi *et al.* 24, Qian *et al.* 24]

- While there have been multiple proposal for implement splittable flows…
  - Examples: MPTCP [Raiciu et al. 2011], packet-spraying [Dixit et al. 2013]

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- While there have been multiple proposal for implement splittable flows…
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- … none of them has been adopted in practice
  - Reasons: require deep changes to current protocols, unverified in large scale

This talk

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- **Question #2:** Is a minimum congestion routing of unsplittable flows computable in polynomial-time? If not, how well can it be approximated?

Prior results on minimum congestion routing

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[Raghavan and Tompson 87, Chakrabarti *et al.* 07]

- In general networks, there are approximation algorithms that ensure worst-case congestion and approximation factor **poly-logarithmic** in size N of the network

Prior results on minimum congestion routing

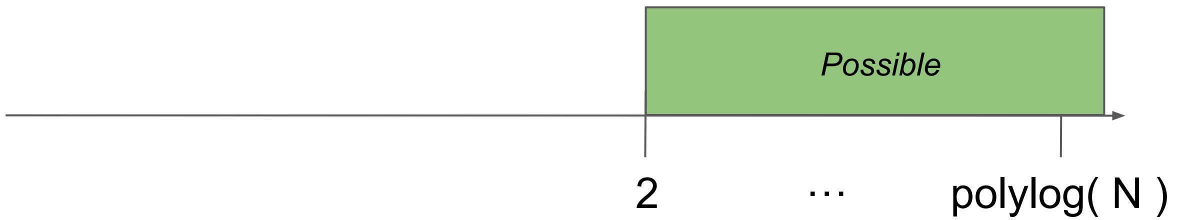
[Al-Fares et al. 08]

- In Clos networks, random ECMP ensures worst-case congestion and approximation factor **poly-logarithmic** in number N of the middle switches

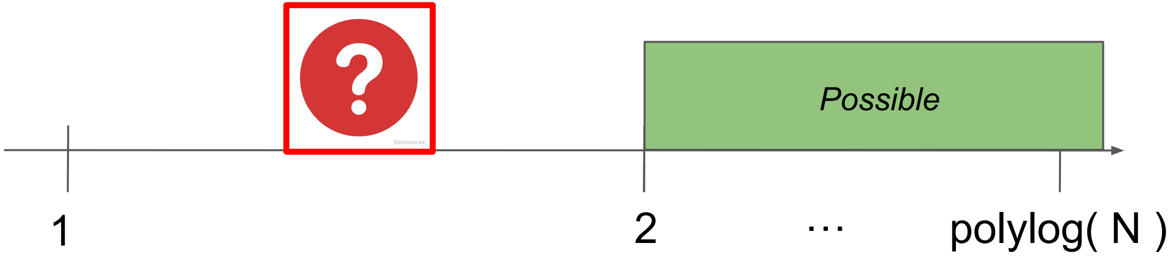
Prior results on minimum congestion routing

[Melen and Turner 89, Al-Fares et al. 10]

- In Clos networks, state-of-the-art heuristics ensure worst-case congestion and approximation factor of **2**

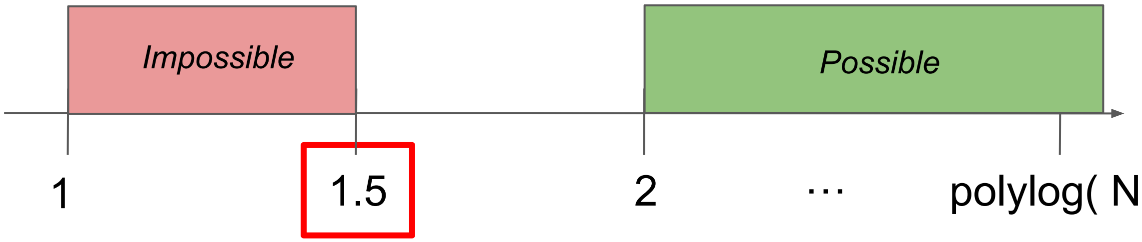


What we know: worst-case congestion and approximation factor is between 1 and 2



What *we show:* worst-case congestion and approximation factor is between 1.5 and …

- **Result #1:** (#1.1) Minimum congestion is **≥ 1.5**. (#1.2) Furthemore, it is NP-hard to approximate minimum by factor **< 1.5**.



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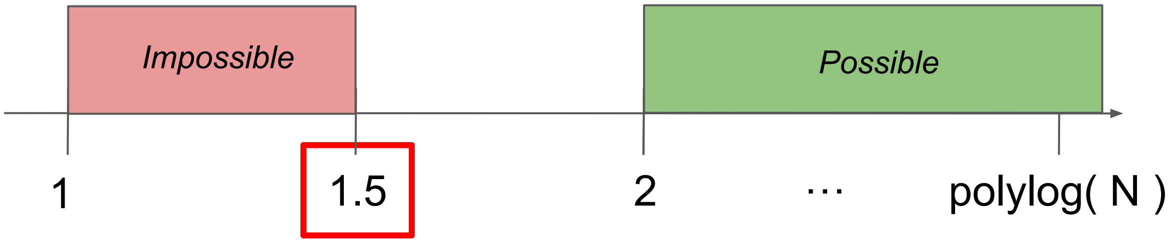
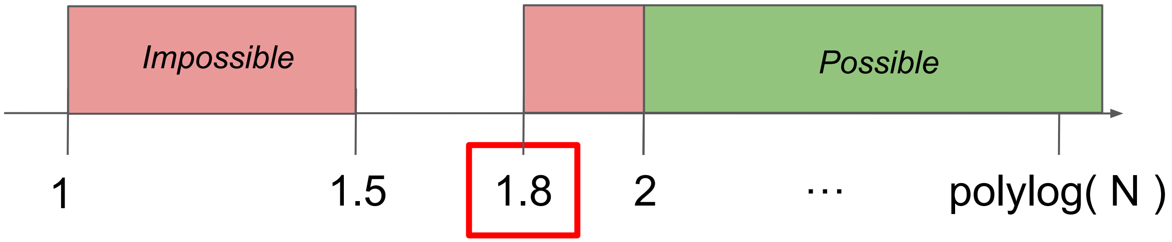
- **Implication:** Special structure of Clos networks cannot avoid some flows obtaining ≤ 2/3 of their demands



… 1.8

- **Result #2:** There is a polynomial-time algorithm that ensures congestion **≤ 1.8** for all sets of flows, and approximates minimum congestion by a factor **≤ 1.8**



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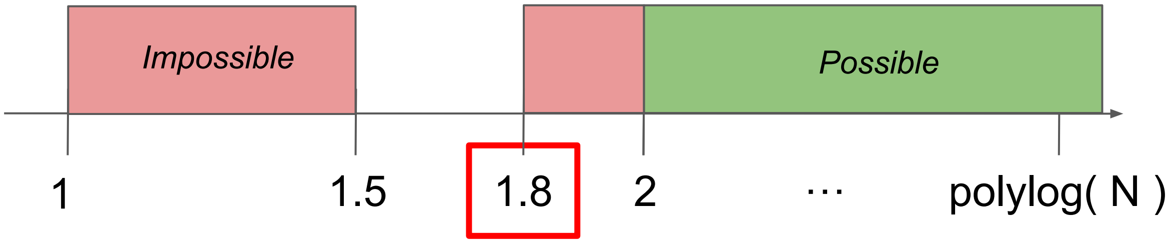
- **Result #2:** There is a polynomial-time algorithm that ensures congestion **≤ 1.8** for all sets of flows, and approximates minimum congestion by a factor **≤ 1.8**

- **Implication:** Known heuristics are not optimal



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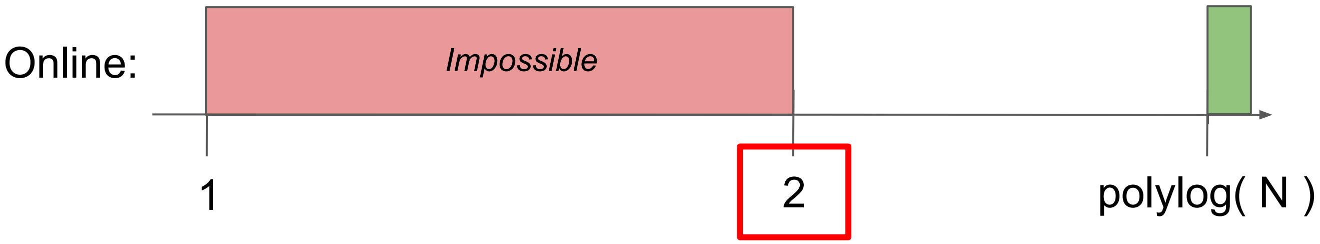
- **Result #3:** No online algorithm (even randomized) approximates minimum congestion by a factor **< 2**



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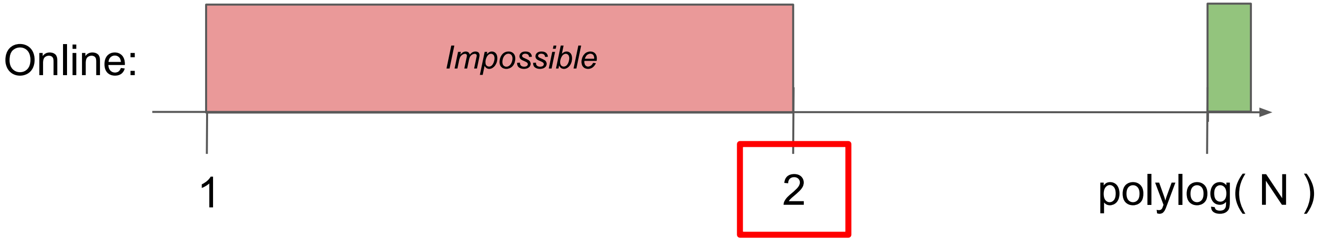
- **Result #3:** No online algorithm (even randomized) approximates minimum congestion by a factor **< 2**
- **Implication:** There is a strict separation between online and offline settings



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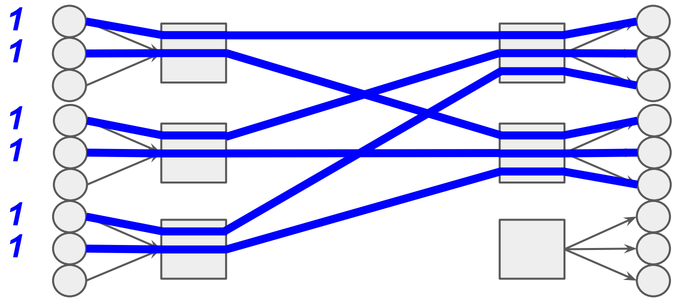
- **Lemma** [Hwang 83]: For all sets of flows with demand *1*, there is a routing with congestion 1, and such a routing can be found in polynomial-time

- **Theorem #1.1**: There is a set of flows with demands *1* or *0.5* such that the minimum congestion is **1.5**

- **Theorem #1.2**: For a set of flows with demands *1* or *0.5*, deciding if minimum congestion is **≤ 1** or **1.5** is NP-complete

Key idea for limits to congestion and approximation

- **Lemma**: For every routing of the tunnel gadget with congestion 1, there is a different middle switch at each input switch to which no flow is assigned



Funnel gadget

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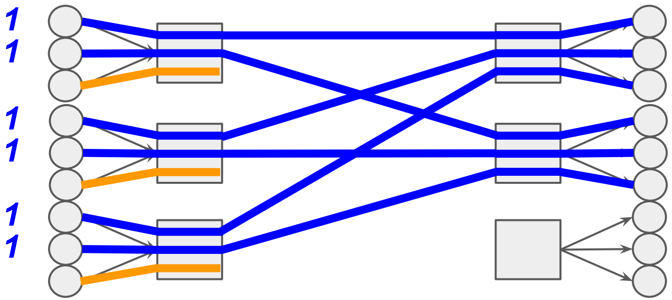
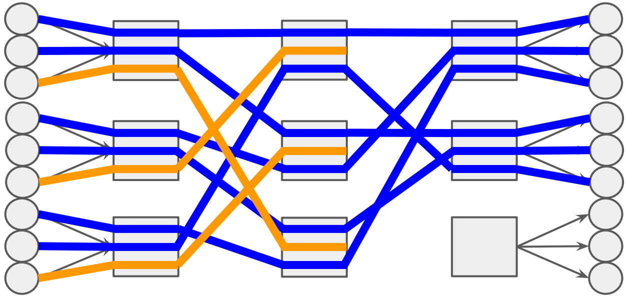


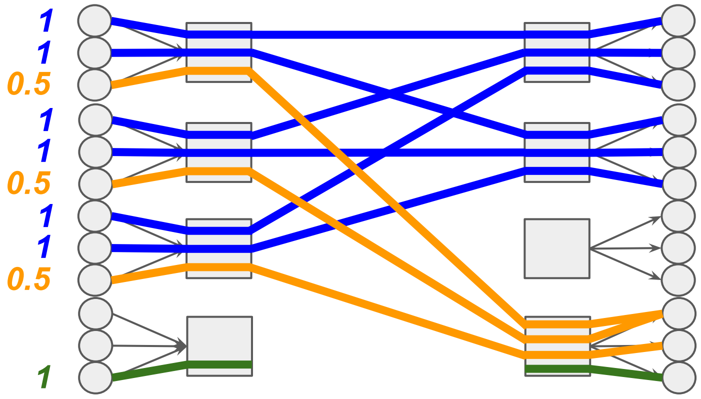
Funnel gadget

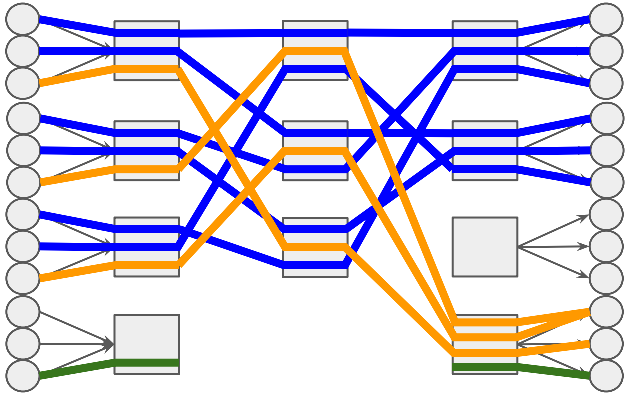


Routing with congestion 1

Proof hint for Theorem #1.1: Add demand 0.5 flows to funnel gadget such that all middle switches blocked

- **Theorem #1.1**: There is a set of flows with demands *1* or *0.5* such that the minimum congestion is **≥ 1.5**





Proof hint for Theorem #1.2: Establish a reduction from the 3-edge coloring problem

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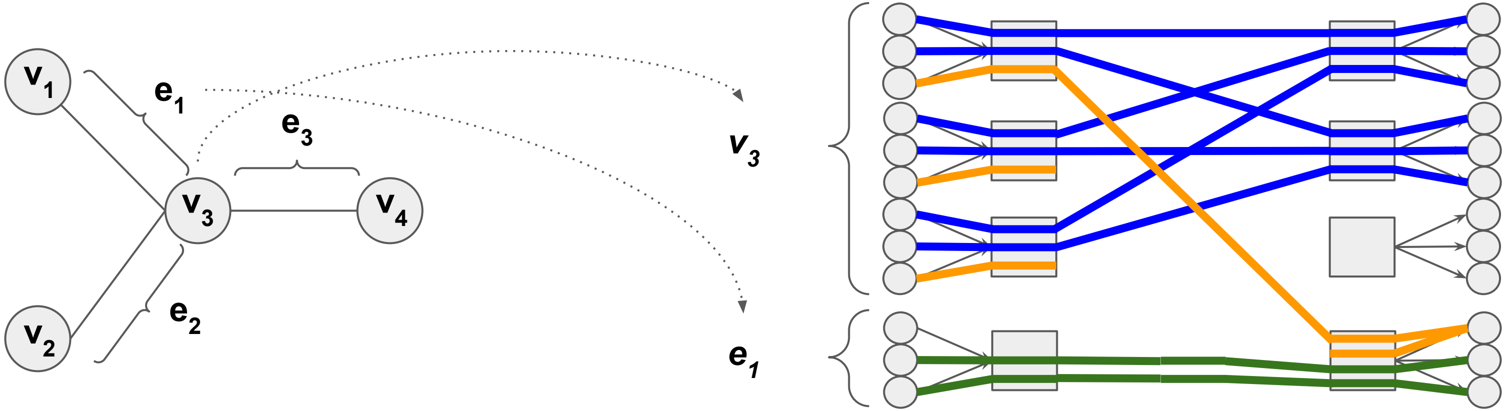

Towards a 1.8-approximation algorithm that ensures congestion 1.8

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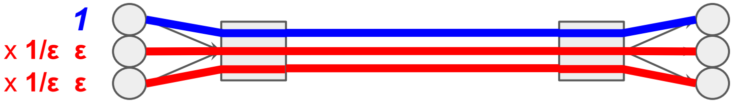
- Prior work suggests two combinatorial algorithms to build upon

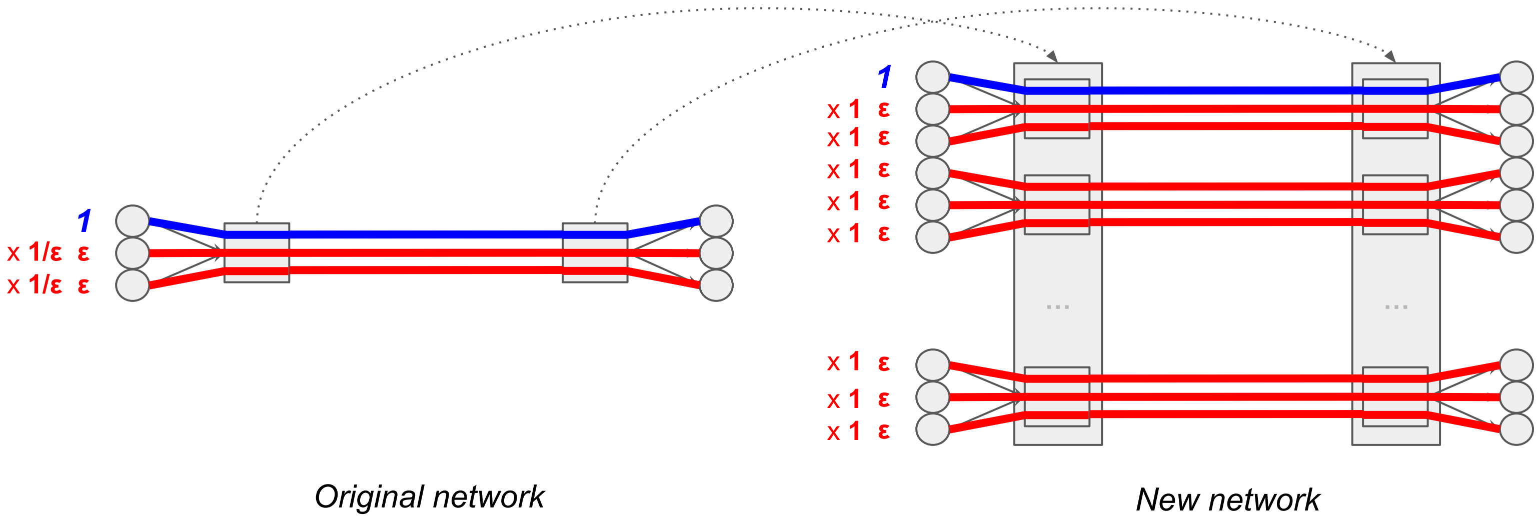
Description of the Melen-Turner algorithm [Melen and Turner 89]



Original network

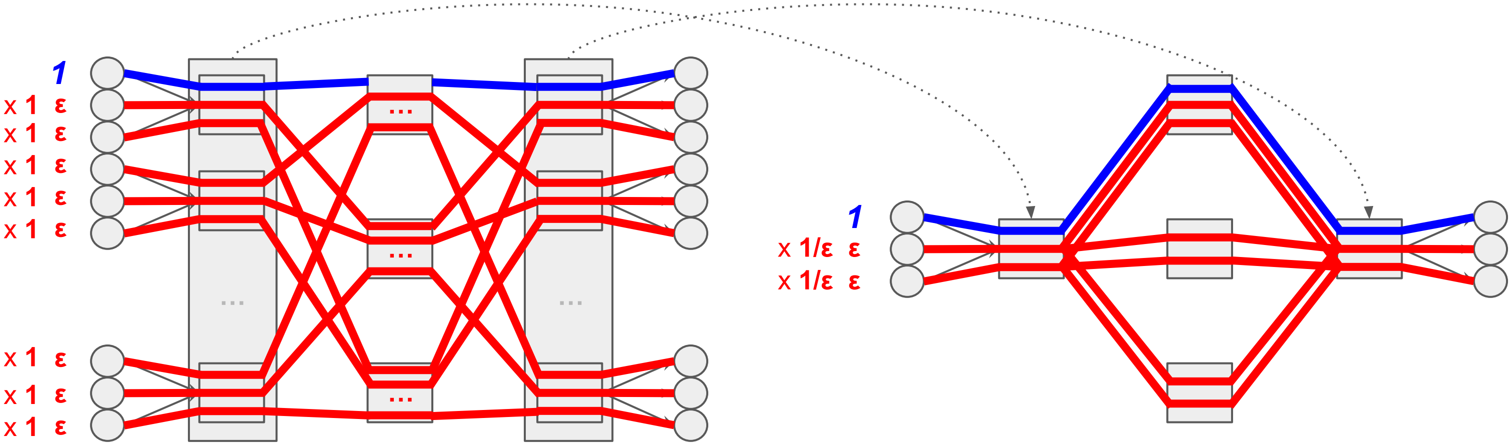
Description of the Melen-Turner algorithm [Melen and Turner 89]

1. Build new network from original one; in new network, there are multiple copies of each ToR, with ≤ N flows per copy in decreasing of demands



Description of the Melen-Turner algorithm [Melen and Turner 89]

2. Find link-disjoint routing in new network

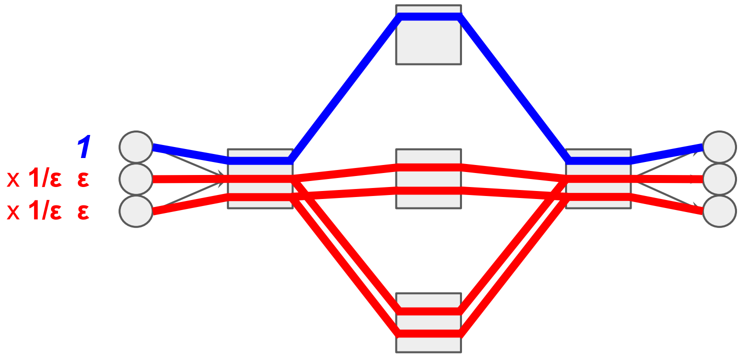


Link disjoint routing in new network

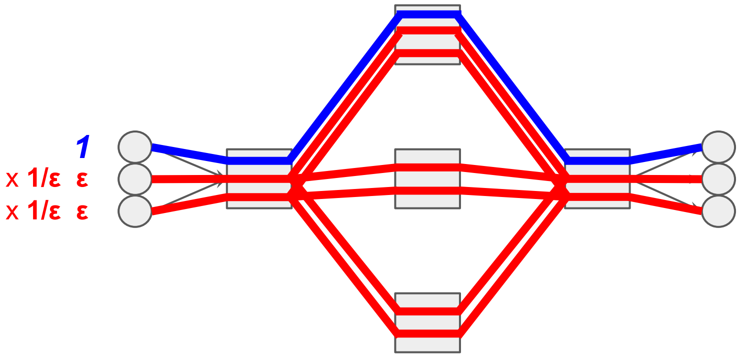
Routing in original network

Melen-Turner algorithm has worst-case congestion and approximation factor no better than 2

- **Lemma:** Melen-Turner algorithm is a **2**-approximation algorithm, and returns a routing with congestion **≤ 2** for all sets of flows; these bounds are tight
- **Tight example:**



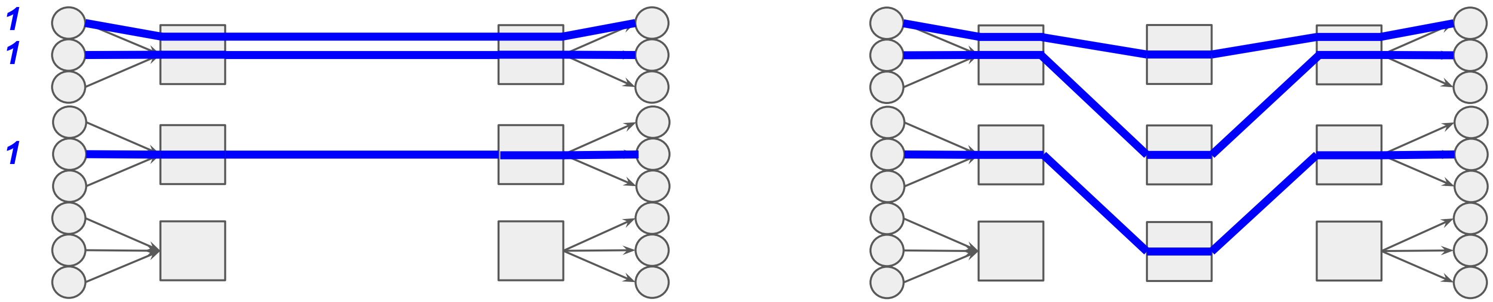
Congestion 1 routing



Congestion 2 routing returned by algorithm

Description of the Sorted-Greedy algorithm [Al-Fares *et al.* 08]

1. Assign flows in decreasing order of demands to minimum congestion paths



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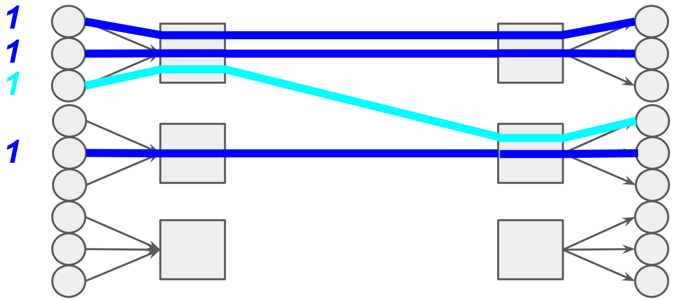
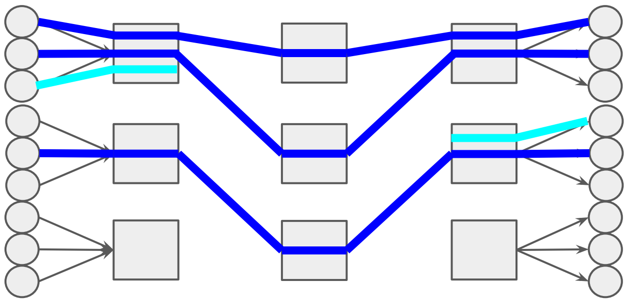


Congestion 1 routing



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  - **Phase 2:** Route remaining flows via Sorted-Greedy algorithm without increasing congestion > C
- **Theorem #2**: If C = 1.8, then algorithm returns a routing with congestion **≤ 1.8** for all sets of flows; however, it is not a **1.8**-approximation algorithm

A 1.8-approximation algorithm that guarantees congestion 1.8

- Two-phase algorithm bridged by threshold C and lower bound L on OPT
  - **Phase 1:** Route a subset of the flows including all with demand > ⅓ x L via Melen-Turner algorithm with congestion ≤ C x L
  - **Phase 2:** Route remaining flows via Sorted-Greedy algorithm without increasing congestion > C x OPT
- **Theorem #2**: If C = 1.8, then algorithm is a **1.8**-approximation algorithm, and returns a routing with congestion **≤ 1.8** for all sets of flows

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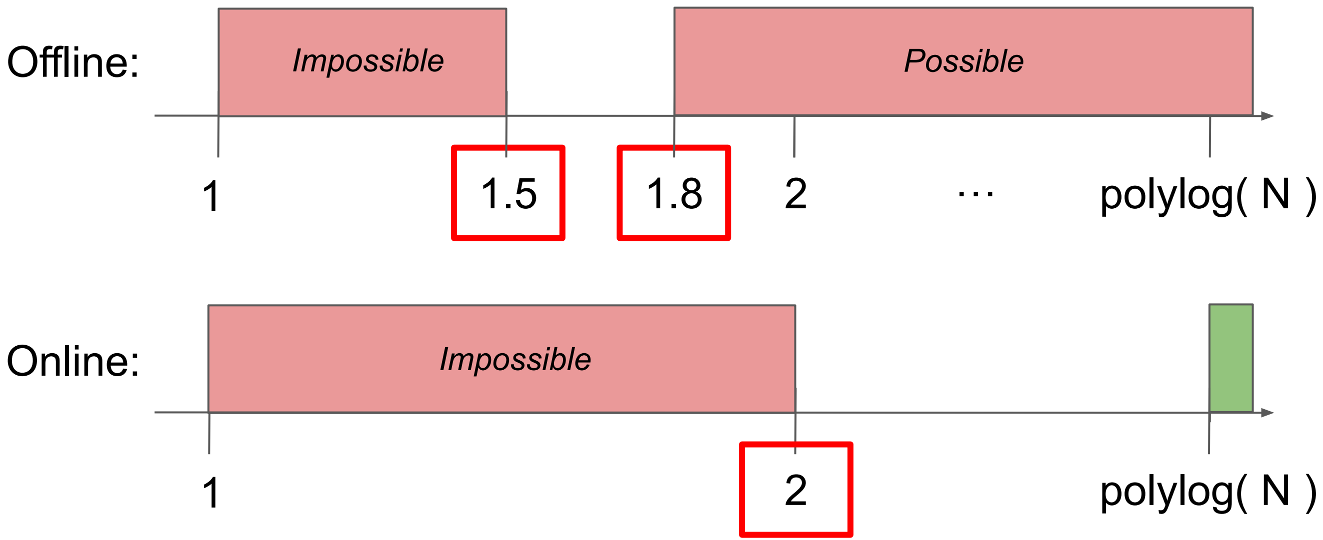
- **Question #2** (Multi-path routing): Does jointly route flows and divide their demands over a *constant* number of paths guarantee a congestion-free network?

Conclusion



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